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A New IDEA Model for QFD with Imprecise Information

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Abstract
Quality Function Deployment (QFD) is a methodology for translating customer requirements (CRs) into relevant engineering design requirements (DRs). It is a team-based and disciplined approach to product design, engineering and production and provides in-depth evaluation of a product. The QFD team is responsible for assessing the relationships between CRs and DRs and the interrelation between DRs. In practice, each member demonstrates significantly different behavior from the others and generates different assessment results, leading to the QFD with uncertainty. In this paper enter each member's subjective assessment into the house of quality (HOQ) and allocate a set imprecise data for each relationship. In order to get the scores (efficiencies) of DRs, we will introduce a new data envelopment analysis methodology which it's inputs and outputs are set imprecise data. In this new methodology, two specially constructed DEA models are used to derive the best efficiency and the lower bound of the best efficiency for each DR. The answers of each model will be a set data efficiencies which by using the Arithmetic Average Method we will reach the goal.

Keywords: Data Envelopment Analysis, Quality Function Deployment, House Of Quality, Set Imprecise Data, Group Decision Making.

1 Introduction

Organizations that pay attention to quality and customer requirements (CRs) stay ahead of competition and survive in the modern competitive market place. A variety of tools are available to organizations in order to help them achieve this goal. Quality function deployment (QFD) [1] is one such extremely important quality management tool that is useful in product design and development and for benchmarking. It has been successfully introduced in many industries to improve design process, customer satisfaction, and to create competitive advantage [1,2]. The quality Function Deployment concept has been first developed in Japan in 1966 by Yoji Akao and disseminated through a paper, in 1972 [3]. It arrived in the United-States in 1984 and later on in other industrial countries [4]. A comprehensive literature review of QFD and it's extensive applications is provided by Chan and Wu [5]. QFD is known by it's house of quality (HOQ) which has a matrix format [1]. HOQ is an important tool for QFD activities, containing information on "what", i.e., customer requirements (CRs), "how", i.e., design requirements (DRs), relationship between "CR" and "DR", and a triangular-shaped matrix placed over the design requirements corresponds to the interrelation between them. Traditionally, QFD rates the design requirements (DRs) with respect to customer requirements (CRs), and aggregates the ratings to get relative importance scores of DRs i.e., traditional QFD uses the weighted sum method to prioritize DRs [6,7]. In practice, a QFD team is set up to determine the importance levels of DRs by computing the relationships between CRs and DRs and relationship between the DRs

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themselves with the importance score of each CR. A group of customers are selected for assessing the relative importance of CRs. In order to get the best relative importance scores of DRs in QFD, this methodology requires a significant number of judgments from QFD teams and customers. There are studies to determine the priority for CRs and DRs in QFD literature. The relative importance of CRs may be obtained using simple methods such as direct rating, or more complex ones such as the swing methods [8], the analytic hierarchy process (AHP) [9], a well-known and commonly used multi-criteria decision making method, and it's variants: fuzzy AHP, analytic network process (ANP)[10] and fuzzy ANP. Design engineers usually do not have sufficient knowledge and information about the influence of engineering responses on CRs, due to the lack of information or language hedge from the customer. These consideration have made the applications of fuzzy approaches and imprecise problems in the importance score of each CR and the relationships between CRs and DRs and the relationships between DRs themselves. There are several studies to deal with this vague nature of QFD [11,12,13]. Fuzzy logic and fuzzy inference have been extensively applied to assess the importance of CRs and prioritize DRs [14,15]. Some studies consider the fact that each member of QFD team may express different ideas. To cope with this situation Ho et al [16] discuss group behaviors in QFD and present an integrated group decision making approach for aggregating team members' opinions in the case where some members in a team have an agreed criteria set while others prefer individual criteria sets. Then they use voting and linear programming techniques they get the relative importance of DRs. Buyukozkan and Feyzioglu [17] present a fuzzy logic based group decision making approach with multiple expression formats for QFD. Chin et al [18] claimed that although there are quite a lot of studies to deal with the ambiguous nature in the process of QFD they do not address the issue of how to deal with incomplete, imprecise and missing information in QFD. To deal with this uncertainty environment, they developed an evidential reason (ER) based QFD methodology. This methodology allows customers and QFD team members to express their opinions using a unified belief structure independently, can accommodate judgments that may be complete or incomplete, precise or imprecise and ignorance information. On the other hand, traditional QFD process does not explicitly incorporate cost and environmental factors. These factors are incorporated in further analysis. Some studies have considered the level of difficulty, some others cost or ease of implementation. However in general, studies have considered only one extra factor in their analysis [19,20]. Finally Ramanathan and Yunfeng [21] proposed data envelopment analysis (DEA) [22,23] to incorporate cost and environmental factors in QFD. They view each DR as a decision making unit (DMU). Then by using the input and output definition in DEA, they classify CRs and other factors as inputs and outputs. So CRs and ease of implementation are considered as outputs, factors like cost and level of difficulty as inputs. If there is no input (like in traditional QFD) they use a dummy input with a constant value of one for all DMUs. By solving the CCR-input oriented model for each DMU (DR) they get the relative importance of each DR. In this paper to cope with the uncertainty in QFD, a new imprecise DEA (IDEA) methodology [24] will be introduced to get the best scores for each DR. In this new methodology we will enter each member's subjective judgments into the HOQ and allocate a set imprecise data for each relationship, then we introduce two especially constructed DEA models which their inputs and outputs are set imprecise data. By solving these two models for each DR we get a set efficiency for each of them. Then by using the arithmetic average method, we will get the best and the worst efficiencies for each DR. Since DEA and

QFD are not new to the field of operations management, interested readers are referred to the prominent articles. The rest of this paper is organized as follows: The use of DEA in estimating the relative importances of DRs are explained in section 2. We introduce a new imprecise DEA methodology and describe in details in section 3. This paper is concluded in section 4.

2 DEA-QFD methodology
2.1 Quality function deployment

QFD begins with the identification of customer requirements and their mapping into relevant engineering design requirements, as shown in Fig. 1, where $CR_1, CR_2, ..., CR_m$ are the m identified customer requirements, $DR_1, DR_2, ..., DR_n$ are the n relevant engineering design requirements, $w_1, w_2, ..., w_m$ are the relevant importances of CRs which $\sum_{i=1}^{m} w_i = 1$ and $w_i > 0$ for i=1,…,m, R_{ij} is the relationship between CR_i and DR_j , and r_{jk} is the interrelationship between DR_j and DR_k , satisfying $r_{jk} = r_{kj}$ for j, k= 1,…,n. The relationship between CRs and DRs reflects the impact of the fulfillment of DRs on the satisfaction of CRs. These relationships should be developed by QFD team members. The relationship between CRs and DRs and the relationship between the DRs themselves are usually determined subjectively by ambiguous or vague judgments. However they are usually captured using symbols converted into crisp numbers using different measurement scales. The degree of these relationships is usually expressed on a scale system such as 0-1-3-9 or 0-1-3-5, representing linguistic expressions such as "no relationship", "weak/possible relationship", "medium/moderate relationship", and "strong relationship". In this paper, rating scale 0-9 is defined to characterize different strengths of the relationships between CRs and DRs as shown in Table 1. Other rating scales can also be defined. It is not our purpose to explore which rating scale is the best or more appropriate for a specific situation, which is beyond the scope of this paper.

As the shape of this figure looks similar to a house, so it referred to as the house of quality.

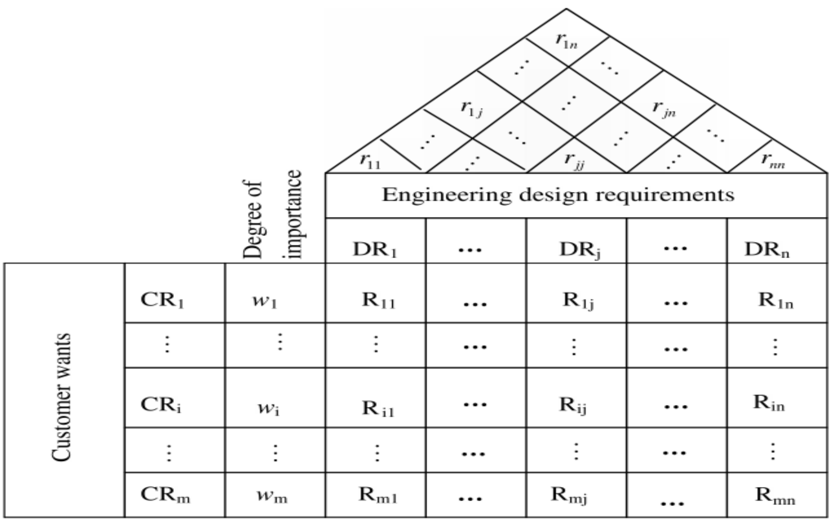


Fig. 1 The house of quality in QFD

Usual procedure for estimating the relative importances of DRs with respect to CRs is to use weighted arithmetic aggregation rule. Note that, when there are significant interrelations between the DRs, in many studies the following normalization procedure suggested byWasserman [6] is usually employed for this purpose:

$$\Re_{ij}^{norm} = \frac{\sum_{l=1}^{N} R_{rl} r_{lj}}{\sum_{j=1}^{N}\sum_{l=1}^{N} R_{rl} r_{lj}}, \quad i = 1,\ldots,m, \quad j = 1,\ldots,N \tag{1}$$

where R_{ij} denotes the relation level in terms of score between the CR_i and DR_j ; and r_{ij} is the interrelation score between DR_i and DR_j . $\Re_{ij}^{norm}$ is the normalized relationship value between CR_i and DR_j , m is the number of CRs, N is the number of DRs and $\sum_j \Re_{ij}^{norm} = 1$ for each i. Thus, if there is significant interrelations between the DRs, $\Re_{ij}^{norm}$ must be used in estimating the relative importances of DRs with respect to CRs, otherwise R_{ij} can be directly used.

Table 1 Rating scales for the relationships between CRs and DRs, and interrelation between DRs themselves

Rating	Definition for	
	Relationship matrix	Matrixinterrelationship
9	Very strong relationship	Very strong interrelation
7	strong relationship	strong interrelation
5	Moderate relationship	Moderate interrelation
3	Weak relationship	Moderate interrelation
1	Very weak relationship	Very weak interrelation
0	No relationship	No interrelation
2,4,6,8	The relationship between these intervals	The interrelation between these intervals

2.2 Incorporating additional factors in QFD using DEA

As mentioned in 2.1, traditional QFD involves a simple procedure for estimating the relative importances of DRs with respect to CRs. However this procedure is disable to estimate the relative importances of DRs with respect to not only CRs but also other additional factors such as cost, ease of implementation, environmental factors, etc. have to be considered. To cope with this weakness, Ramanathan and Yunfeng employ DEA to compute the relative importances of DRs when several additional factors need to be considered. They prove that the relative importance values computed by DEA coincide with traditional QFD calculations when only the ratings of DRs with respect to CRs are considered, and when only one additional factor (such as cost) is considered. They show that DEA provides a simple and general framework facilitating QFD computations when more factors need to be considered, and has the flexibility to treat QFD ratings as qualitative factors [21].

In this methodology each DR is considered as a DMU and the efficiency score of DR is considered as a measure of its relative importance. In order to classify the CRs and other additional factors as inputs and outputs, they use the suggestion of Golanyand Roll [25]. By using this logic, CRs and factors such as ease of implementation are considered as outputs, while factors such as cost and level of difficulty are considered as inputs. The corresponding output-input matrices are shown in Table 2.

Table 2 Classification of CRs and additional factors as inputs and outputs

	Output1(CR1)	Output1(CR1)	⋮	Output1(CR1)	Additional output factors			Additional input factors		
					Outputk+1	⋮	Output s	Input 1	⋮	Input m
DMU1(DR1)	y_{11}	y_{21}	⋮	y_{k1}	$y_{k+1,1}$	⋮	y_{s1}	x_{11}	⋮	x_{m1}
DMU1(DR1)	y_{12}	y_{22}	⋮	y_{k2}	$y_{k+1,2}$	⋮	y_{s2}	x_{12}	⋮	x_{m2}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
DMUn(DRn)	y_{1n}	y_{2n}	⋮	y_{kn}	$y_{k+1,n}$	⋮	y_{2n}	x_{1n}	⋮	x_{mn}

In order to impose the relative importances of CRs, the method of Assurance region (AR) is employed. Thus additional constraints that specify the relationships among the multipliers are appended to the DEA model. Hence, each DMU has m inputs and s outputs (which k of them are the CRs (k<s)), based on which the following restricted input-oriented CCR model is built to assess the efficiency score (relative importance) of DRs:

$$\begin{aligned}
\max \quad & E_0 = \sum_{r=1}^{s} u_r y_{r0} \\
s.t. \quad & \\
& \sum_{i=1}^{m} v_i x_{i0} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1,\dots,n \\
& u_r \ge 0, \quad v_i \ge 0, \quad r = 1,\dots,s, \quad i = 1,\dots,m
\end{aligned} \tag{2}$$

By imposing the importance of CRs and other additional factors by introducing additional constraints of the form of $u_r = d_r u_1$ for all $r = 1,\dots,s$ (which $r = 1,\dots,k$) indicates the importances of CRs, the model can be rewritten as follows:

$$\begin{aligned}
\max \quad & E_0 = \sum_{r=1}^{s} u_r y_{r0} \\
s.t. \quad & \\
& \sum_{i=1}^{m} v_i x_{i0} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1,\dots,n \\
& u_r = d_r u_1, \qquad r = 1,\dots,k \\
& u_r \ge 0, \quad v_i \ge 0, \quad r = 1,\dots,s, \quad i = 1,\dots,m
\end{aligned} \tag{3}$$

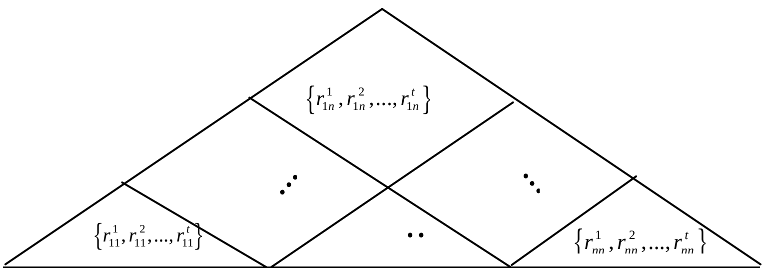
The linear programming model (3) is solved for all the DMUs to estimate their relative scores.

3 New IDEA methodology

Due to the usual procedure in traditional QFD, the relationship between CRs and DRs and interrelationship between DRs themselves should be determined by QFD team members. Different from the traditional QFD, which requires team members to provide a consensus assessment for each relationship, we allow the team members to enter their opinions into the calculation, directly. Suppose that QFD team consists of t members or decision makers (DM): $DM_1, DM_2, ..., DM_t$ who judged that the relationship between CR_r (or other additional output factors) and DR_j are $y_{rj}^1, y_{rj}^2, ..., y_{rj}^t$, respectively and the relationship between cost (or other additional input factor) and DR_j are $x_{ij}^1, x_{ij}^2, ..., x_{ij}^t$, respectively, and the interrelation between DRs themselves are $r_{kj}^1, r_{kj}^2, ..., r_{kj}^t$, respectively so that y_{rj}^1, x_{ij}^1 and r_{kj}^1 that l= 1,…,t belong to the rating scale that exists in Table 1.

Thus the relationship between $output_r$ and DR_j is $\{y_{rj}^1, y_{rj}^2, ..., y_{rj}^t\}$ that is a set output data and the relationship between Input$_i$ and DR_j is $\{x_{ij}^1, x_{ij}^2, ..., x_{ij}^t\}$ that is a set input data and the interrelation between DR_k and DR_j is $\{r_{kj}^1, r_{kj}^2, ..., r_{kj}^t\}$ that is a set interrelation data.

If we enter these set data into HOQ, the HOQ will transform into a new HOQ with imprecise data, as shown in Fig. 2. We called it IHOQ (Imprecise House Of Quality).



		DR_1	$\cdots$	DR_n
CR_1	$\{w_1^1, w_1^2, ..., w_1^t\}$	$\{R_{11}^1, R_{11}^2, ..., R_{11}^t\}$	$\cdots$	$\{R_{1n}^1, R_{1n}^2, ..., R_{1n}^t\}$
CR_2	$\{w_2^1, w_2^2, ..., w_2^t\}$	$\{R_{21}^1, R_{21}^2, ..., R_{21}^t\}$	$\cdots$	$\{R_{2n}^1, R_{2n}^2, ..., R_{2n}^t\}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
CR_m	$\{w_m^1, w_m^2, ..., w_m^t\}$	$\{R_{m1}^1, R_{m1}^2, ..., R_{m1}^t\}$	$\cdots$	$\{R_{mn}^1, R_{mn}^2, ..., R_{mn}^t\}$

Fig. 2 IHOQ: House Of Quality with Interval information

If we enter these set data into a DEA model, the model will be nonlinear. In order to transform it into a linear programming, equivalently, we employ some assumptions: In order to make an ordering set data, without loss of generality, we assume that this set is non-decreasing, otherwise we order them. We represent each element of the set by y_{rj}^{l} for outputs and x_{ij}^{l} for inputs which $l = 1,...,t$ so that $l = \{1,...,t\}$.

Different set of x_{ij}^{l}, y_{rj}^{l} have different value of efficiency that evaluating each of them is so expensive and time-consuming, so here we evaluate, only, the best and the lower bound of the best efficiencies. In order to reach our goal, we introduce a new pair of IDEA models.

3.1 Proposed models

We construct two especial IDEA models as follows.
Suppose there are n DRs that we must estimate the relative importances of them with respect to s output factors and m input factors, and suppose $\beta = \alpha - m$ and

$$k_1 = \begin{cases} 1 & 1 \le \alpha \le m \\ 0 & otherwise \end{cases}, \qquad k_2 = \begin{cases} 1 & m+1 \le \alpha \le m+s \\ 0 & otherwise \end{cases}$$

If there is no significant interrelation between DRs, we can construct the following linear models for calculating the set efficiencies of DR_o:

$$\begin{aligned} \max \quad & Eff_{\alpha 0}^{U} = \sum_{r=1, r\neq\beta}^{s} u_r y_{r0}^{t} + \frac{1}{t} k_2 \sum_{l=1}^{t} u_\beta y_{\beta 0}^{l} \\ s.t. \quad & \\ & \sum_{i=1, i\neq\alpha}^{m} v_i x_{i0}^{1} + \frac{1}{t} k_1 \sum_{l=1}^{t} v_\alpha x_{\alpha 0}^{l} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj}^{t} - \sum_{i=1}^{m} v_i x_{ij}^{1} \le 0, \quad j = 1,...,n \\ & u_r \ge \varepsilon, \quad v_i \ge \varepsilon, \quad r = 1,...,s, \quad i = 1,...,m \end{aligned} \tag{4}$$

$$\begin{aligned} \max \quad & Eff_{\alpha 0}^{U} = \sum_{r=1, r\neq\beta}^{s} u_r y_{r0}^{1} + \frac{1}{t} k_2 \sum_{l=1}^{t} u_\beta y_{\beta 0}^{l} \\ s.t. \quad & \\ & \sum_{i=1, i\neq\alpha}^{m} v_i x_{i0}^{t} + \frac{1}{t} k_1 \sum_{l=1}^{t} v_\alpha x_{\alpha 0}^{l} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj}^{t} - \sum_{i=1}^{m} v_i x_{ij}^{1} \le 0, \quad j = 1,...,n \\ & u_r \ge \varepsilon, \quad v_i \ge \varepsilon, \quad r = 1,...,s, \quad i = 1,...,m \end{aligned} \tag{5}$$

where subscript 0 represents the design requirements (DMU) under evaluation, u_r and v_i are decision variables and α is the non-Archimedean infinitesimal. α is the index of set input or set output of DR (DMU in DEA) under consideration. On the other words if a factor of set data (set input or set output) is under consideration for 0^{th} DMU, except it, other set data of 0^{th} DMU in model (5) and (4) will be in their highest values and their lowest values, respectively.

If the relevant importances of CRs are precise weights, we can impose them into our proposed model by using AR method in DEA. So the models will be as follows:

$$
\begin{aligned}
\max \quad & Eff_{\alpha 0}^{U} = \sum_{r=1, r\neq\beta}^{s} u_r y_{r0}^{t} + \frac{1}{t} k_2 \sum_{l=1}^{t} u_\beta y_{\beta 0}^{l} \\
s.t. \quad & \\
& \sum_{i=1, i\neq\alpha}^{m} v_i x_{i0}^{1} + \frac{1}{t} k_1 \sum_{l=1}^{t} v_\alpha x_{\alpha 0}^{l} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj}^{t} - \sum_{i=1}^{m} v_i x_{ij}^{1} \leq 0, \qquad j = 1,\ldots,n \\
& u_r = d_r u_1, \qquad r = 1,\ldots,k \\
& u_r \geq \varepsilon, \quad v_i \geq \varepsilon, \quad r = 1,\ldots,s, \quad i = 1,\ldots,m
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
\max \quad & Eff_{\alpha 0}^{U} = \sum_{r=1, r\neq\beta}^{s} u_r y_{r0}^{1} + \frac{1}{t} k_2 \sum_{l=1}^{t} u_\beta y_{\beta 0}^{l} \\
s.t. \quad & \\
& \sum_{i=1, i\neq\alpha}^{m} v_i x_{i0}^{t} + \frac{1}{t} k_1 \sum_{l=1}^{t} v_\alpha x_{\alpha 0}^{l} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj}^{t} - \sum_{i=1}^{m} v_i x_{ij}^{1} \leq 0, \qquad j = 1,\ldots,n \\
& u_r = d_r u_1, \qquad r = 1,\ldots,k \\
& u_r \geq \varepsilon, \quad v_i \geq \varepsilon, \quad r = 1,\ldots,s, \quad i = 1,\ldots,m
\end{aligned} \tag{7}
$$

where $Eff_{\alpha 0}^{U}$ in (6) for $\alpha = 1,\ldots,m+s$, stands for the best possible set relative efficiency achieved by DMU_o when all the DMUs are in the state of best production activity [26], while $Eff_{\alpha 0}^{L}$ in (7) for $\alpha = 1,\ldots,m+s$, stands for the lower bound of the best possible set relative efficiency of DMU_0.

Proposed models enable us to find the max and min set efficiency of the best possible set relative efficiency of DR_0. By taking the average of the max set efficiencies (min set efficiencies) we can get the best (the lower bound of the best) efficiency of each DR.

Definition 3.1

$$\theta_0^U = \frac{1}{m+s} \sum_{\alpha=1}^{m+s} Eff_{\alpha 0}^{U} \tag{8}$$

$$\theta_0^L = \frac{1}{m+s} \sum_{\alpha=1}^{m+s} Eff_{\alpha 0}^{L} \tag{9}$$

Where θ_0^U in (8), represents the best possible relative efficiency of $D\mathrm{MU}_o$ and θ_0^L in (9) represents the lower bound of the best possible relative efficiency of DMU_0. They constitute a possible best relative efficiency interval $\left[\theta_0^L, \theta_0^U\right]$ for DR_o.

About the relationship between θ_0^L and θ_0^U we have the following theorem.

Theorem 3.2

If $Eff_{\alpha 0}^{U^*}$ and $Eff_{\alpha 0}^{L^*}$ for $\alpha = 1, \dots, m+s$ are the optimum objective function values of models (6) and (7), respectively, then

$Eff_{\alpha 0}^{L^*} \le Eff_{\alpha 0}^{U^*}$.

Proof.

Suppose u_1^* and v_i^* are the optimal solution to model (7). For $1 \le \alpha \le m$, let

$$\varphi_0 = \sum_{i=1, i\neq\alpha}^{m} v_i^* x_{i0}^1 + \frac{1}{t} k_1 \sum_{l=1}^{t} v_\alpha^* x_{\alpha 0}^l$$

$$\hat{u}_r = \frac{u_r^*}{\varphi_0}, \quad r = 1, \dots, s, \quad \hat{v}_i = \frac{v_i^*}{\varphi_0}, \quad i = 1, \dots, m$$

It follows that

$$\varphi_0 = \sum_{i=1, i\neq\alpha}^{m} v_i^* x_{i0}^* + \frac{1}{t} k_1 \sum_{l=1}^{t} v_\alpha^* x_{\alpha 0}^l \le \sum_{i=1, i\neq\alpha}^{m} v_i^* x_{i0}^t + \frac{1}{t} k_1 \sum_{l=1}^{t} v_\alpha^* x_{\alpha 0}^l = 1,$$

$$\sum_{i=1, i\neq\alpha}^{m} \hat{v}_i x_{i0}^1 + \frac{1}{t} k_1 \sum_{l=1}^{t} \hat{v}_\alpha x_{\alpha 0}^l = \sum_{i=1, i\neq\alpha}^{m} \frac{v_i^*}{\varphi_0} x_{i0}^1 + k_1 \sum_{l=1}^{t} \frac{v_\alpha^*}{t\varphi_0} x_{i0}^l = \frac{1}{\varphi_0}\left(\sum_{i=1, i\neq\alpha}^{m} v_i^* x_{i0}^1 + \frac{1}{t} k_1 \sum_{l=1}^{t} v_\alpha^* x_{\alpha 0}^l \right) = 1,$$

$$\sum_{r=1}^{s} \hat{u}_r y_{rj}^t - \sum_{i=1}^{m} \hat{v}_i x_{ij}^1 = \frac{1}{\varphi_0}\left(\sum_{r=1}^{s} u_r^* y_{rj}^t - \sum_{i=1}^{m} v_i^* x_{ij}^1 \right) \le 0, \quad j = 1, \dots, n$$

$$\hat{u}_r = \frac{u_r^*}{\varphi_0} \ge \frac{\varepsilon}{\varphi_0} \ge \varepsilon, \quad \hat{v}_i = \frac{v_i^*}{\varphi_0} \ge \frac{\varepsilon}{\varphi_0} \ge \varepsilon.$$

It is obvious that $\hat{u}_1$ and $\hat{v}_1$ are a feasible solution to model (6). Thus, we have

$$\sum_{r=1}^{s} \hat{u}_r y_{r0}^t \le Eff_{\alpha 0}^{U^*},$$

$$Eff_{\alpha 0}^{L^*} = \sum_{r=1, r\neq\beta}^{s} u_r^* y_{r0}^1 \le \sum_{r=1, r\neq\beta}^{s} u_r^* y_{r0}^t = \sum_{r=1, r\neq\beta}^{s} (\varphi_0 \hat{u}_r) y_{r0}^t$$

$$\varphi_0 \left(\sum_{r=1, r\neq\beta}^{s} \hat{u}_r y_{r0}^t \right) \le \varphi_0 Eff_{\alpha 0}^{U^*} \le Eff_{\alpha 0}^{U^*}$$

In a similar vein, the theorem is proved when $m+1 \le \alpha \le m+s$.

Corollary 3.3

If θ_0^U in (8) and θ_0^L in (9) are the best possible relative efficiency and the lower bound of the best possible relative efficiency of DMU_0, respectively, then

$$\theta_0^L \leq \theta_0^U .$$

Note that, if there is significant interrelation between DRs, in order to enter the impact of these relations on the relationships between CRs and DRs, before ordering the sets, we use the following model.

Definition 3.4

Suppose there are t decision makers (DMs) or team members and let $h_l > 0 (l = 1,...,t)$ be the relative importance weight of each DM satisfying $\sum_{l=1}^{t} h_l = 1$, thus

$$\Re_{ij}^{n} = \sum_{l=1}^{N} h_l R_{ip}^{l}.r_{pj}^{l} , \quad i = 1,...,k , \quad l = 1,...,t , \quad j = 1,...,N$$

where t denotes the number of decision makers, k and stand for the number of CRs and DRs, respectively.

As you see, we do not use any normalization in our procedure. In our view, the purpose of normalization in QFD is to eliminate the dimension of different DRs and make them comparable within the same range of variation such as [0,1]. According to DEA methodology, DEA models deal with different inputs and different outputs with different dimensions and also, the relative importances determined by model (6) and (7) are all within the interval [0,1], Thus these two features make normalization in fact not necessary.

Thus, when there are important interrelations between DRs, we use definition 3.4, then we will enter $\Re_{\mathrm{ij}}^{l}$ (where l=1,…,t) into our proposed models (6) and (7).

We construct proposed models by using the CCR-ε model and in order to take account the relative importances of CRs, we use Assurance Region (AR) method in DEA. By solving these two models for each DR we can get the best and the lower bound of the best set efficiencies of them.

Proposed models enable us to find the max and min set efficiencies of DRs. By taking the average of the max set efficiencies (min set efficiencies) we can get the best (the lower bound of the best) efficiency of each DR.

3.2 Prioritizing the design requirements

For comparing and ranking the interval numbers we use the approach of Wang et al [27]. The approach is summarized as follows.

Let $a = [a_L , a_U]$ and $b = [b_L , b_U]$ be two interval numbers

The degree of preference for interval numbers can be defined as follow.

Definition 3.5

The degree of preference of a over b or (a>b) and b over a or (b>a) is defined as

$$P\left(a>b\right)=\frac{\max(0,a_U-b_L)-\max(0,a_L-b_U)}{(a_U-a_L)+(b_U-b_L)}$$

$$P\left(a>b\right)=\frac{\max(0,b_U-a_L)-\max(0,b_L-a_U)}{(a_U-a_L)+(b_U-b_L)}$$

It is obvious that P(a>b)+P(b>a)=1

Definition 3.6

If P(a>b)>P(b>a), then a is said to be superior to b to the degree of P(a>b), denoted by $a \overset{P(a>b)}{\succ} b$; if P(a>b)=P(b>a)=0.5, then a is said to be indifferent to b denoted by a=b; if P(b>a)>P(a>b), then a is said to be inferior to b to the degree of P(b>a), denoted b $a \overset{P(a>b)}{\prec} b$.

By using this approach the interval numbers ranked and so our DRs will be ranked.

3.3 A numerical illustrative example

In this section we present an illustrative example to show how our proposed IDEA methodology can be used to model uncertainty in QFD. Suppose there are 5 CMs and 5 DMs. The relative importances of CRs are presented in Table 3.

Table 3 The relative importances of CRs

CRs	The relative importances of CRs
CR1	0.2865
CR2	0.0882
CR3	0.2342
CR4	0.1886
CR5	0.2247

Table 4 and 5 show the assessment information provided by 5 DMs on the relationships between the 5 CRs and the 5 DRs and the relationships between the additional factors (cost, ease of implementation and adverse environmental impact) and DRs, respectively. Suppose there is no significant interrelationship between DRs.

Table 4 Assessment on the relationships between the 5 CRs and 5 DRs

CRs	DMs	DRs				
		DR1	DR2	DR3	DR4	DR5
CR1	DM1	9	3	0	1	8
	DM2	8	3	4	1	8
	DM3	9	3	4	1	9
	DM4	8	3	4	1	7
	DM5	7	3	4	1	9
CR2	DM1	0	2	9	9	0
	DM2	1	1	9	9	1
	DM3	1	1	8	9	1
	DM4	0	0	9	9	0
	DM5	0	3	9	9	0
CR3	DM1	1	1	3	5	0
	DM2	1	1	2	4	1
	DM3	1	2	2	3	1
	DM4	0	3	2	5	1
	DM5	1	3	3	5	1

CRs	DMs	DRs				
		DR1	DR2	DR3	DR4	DR5
CR4	DM1	1	0	1	5	1
	DM2	1	0	1	6	1
	DM3	1	0	1	6	1
	DM4	1	0	1	7	0
	DM5	0	1	1	8	1
CR5	DM1	1	0	1	1	1
	DM2	1	0	0	1	1
	DM3	1	0	0	1	1
	DM4	1	0	0	1	1
	DM5	1	0	1	1	1

Table 5 Assessment on the relationships between the additional factors and DRs

additional factors	DMs	DRs				
		DR1	DR2	DR3	DR4	DR5
ease of implementation	DM1	5	1	8	1	6
(output factor)	DM2	7	2	9	1	6
(9 Easy, 1Tough)	DM3	6	0	9	1	6
	DM4	6	1	8	1	7
	DM5	6	1	9	1	6
cost (in cost units)	DM1	4	5	6	10	4
(input factor)	DM2	4	5	7	11	4
	DM3	5	6	6	11	5
	DM4	5	6	7	10	4
	DM5	5	5	6	10	5
adverse environmental impact	DM1	1	7	9	1	1
(input factor)	DM2	1	7	9	1	1
(9 bad, 1 good)	DM3	1	6	9	0	1
	DM4	1	7	9	1	1
	DM5	1	7	9	0	1

Table 6 Assessment on the relationships between all factors and DRs as ordering set data

Factors	DR1	DR2	DR3	DR4	DR5
CR1	{7,8,8,9,9}	{3,3,3,3,3}	{0,1,1,1,1}	{1,1,1,1,1}	{7,8,8,9,9}
CR2	{0,0,0,1,1}	{0,1,1,2,3}	{8,9,9,9,9}	{9,9,9,9,9}	{0,0,0,1,1}
CR3	{0,1,1,1,1}	{1,1,2,3,3}	{2,2,2,3,3}	{3,4,5,5,5}	{0,1,1,1,1}
CR4	{0,1,1,1,1}	{0,0,0,0,1}	{1,1,1,1,1}	{5,6,6,7,8}	{0,1,1,1,1}
CR5	{1,1,1,1,1}	{0,0,0,0,0}	{0,0,0,1,1}	{1,1,1,1,1}	{1,1,1,1,1}
ease of implementation	{5,6,6,6,7}	{0,1,1,1,2}	{8,8,9,9,9}	{1,1,1,1,1}	{6,6,6,6,7}
Cost	{4,4,5,5,5}	{5,5,5,6,6}	{6,6,6,7,7}	{10,10,10,11,11}	{4,4,4,5,5}
adverse environmental impact	{1,1,1,1,1}	{6,7,7,7,7}	{9,9,9,9,9}	{0,0,1,1,1}	{1,1,1,1,1}

Table 6 represents the relationships between all factors and DRs as ordering set data.

Using data from Table 6 and solving our proposed models (6) and (7), for $\alpha = 1,...,8$ (m=2, s= 6) and $\varepsilon = 10^{-10}$, we get the max set efficiencies and the min set efficiencies of DRs. The results are represented in Table 7 and Table 8, respectively.

Table 7 The best possible set relative efficiency of DRs using proposed model (3.3)

α	DR1	DR2	DR3	DR4	DR5
α = 1	0.9915	0.4504	0.8036	0.9615	0.9943
α = 2	1	0.4865	0.8571	0.7943	1
α = 3	1	0.4865	0.8571	1	1
α = *4*	1	0.4524	0.8571	1	1
α = 5	1	0.4299	0.8571	1	1
α = *6*	1	0.45	0.8571	1	0.9859
α = 7	0.9859	0.4865	0.8571	1	0.9859
α = *8*	0.8927	0.4865	0.8190	1	1

Table 8 The lower bound of the best possible set relative efficiency of DRs using proposed model (3.4)

α	DR1	DR2	DR3	DR4	DR5
$\alpha = 1$	0.7082	0.2444	0.7143	0.5031	0.8223
$\alpha = 2$	0.7042	0.22	0.6531	0.5448	0.8451
$\alpha = 3$	0.7042	0.22	0.6531	0.4834	0.8451
$\alpha = 4$	0.7042	0.2448	0.6531	0.4834	0.8451
$\alpha = 5$	0.7042	0.2671	0.6531	0.5371	0.8451
$\alpha = 6$	0.7042	0.2264	0.6531	0.5267	0.8333
$\alpha = 7$	0.7042	0.22	0.6531	0.4834	0.8333
$\alpha = 8$	0.8451	0.22	0.7020	0.4834	0.8857

Now in order to get the best and the lower bound of the best efficiencies and make interval efficiency for each DR, we use arithmetic average.

The final interval efficiencies of DRs are presented in Table 9, from which the ranking order of 5 DRs can be generated as

$$DR5 \overset{68.85\%}{\succ} DR1 \overset{71.99\%}{\succ} DR3 \overset{52.91\%}{\succ} DR4 \overset{100\%}{\succ} DR2$$

using definitions (8) and (9).

Table 9 The interval efficiencies of DRs and their ranking order

Design requirements	DR1	DR2	DR3	DR4	DR5
interval efficiency	[0.7223125,0.9837625]	[0.232825,0.4660875]	[0.6668625,0.84565]	[0.5056625,0.969475]	[0.848125,0.9957625]
ranking order	2	5	3	4	1

As can be seen from Table 9, the proposed methodology evaluate DR5 as the most important design requirements, and DR2 as the least important design requirement.
As mentioned in the previous sections, we enter each subjective judgments of DMs into the HOQ, directly, and considered them as set data for each relationship. We consider all the DM's information together rather than distinct information. As can be seen the answers are logical and the proposed model is sensitive.

4 Conclusion

Organizations that pay attention to quality and customer requirements stay ahead of competition and survive in the modern competitive market place. In this way, QFD is an extremely important quality management tool that is useful in product design and development and for benchmarking. QFD has two important weaknesses. In this paper to cope with these weaknesses we introduced two specially constructed IDEA models in set data environments. By solving these models for each DR we could be able to get the best efficiencies and the lower bound of the best efficiencies of them.

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